① $4^x - 3 \cdot 2^{x+1} + 8 = 0$

$(2^2)^x - 3 \cdot 2^x \cdot 2 + 8 = 0$ $\qquad (2^x)^2 - 3 \cdot 2 \cdot 2^x + 8 = 0$

$(2^x)^2 - 6 \cdot 2^x + 8 = 0 \xrightarrow{2^x = z} z^2 - 6z + 8 = 0$

$$z = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}$$

$$z_1 = \frac{6+2}{2} = 4 \longrightarrow 2^x = 4$$

$$z_2 = \frac{6-2}{2} = 2 \longrightarrow 2^x = 2$$

$\boxed{x_1 = 2 \qquad x_2 = 1}$

② $\begin{cases} x^2 - y^2 = 11 \\ \log x - \log y = 1 \end{cases}$ $\qquad \begin{cases} x^2 - y^2 = 11 \\ \log \frac{x}{y} = \log 10 \end{cases}$

$\begin{cases} x^2 - y^2 = 11 \\ \frac{x}{y} = 10 \longrightarrow x = 10y \end{cases}$ $\rightarrow (10y)^2 - y^2 = 11$

$100y^2 - y^2 = 11$

$99y^2 = 11 \longrightarrow y^2 = \frac{11}{99} = \frac{1}{9}$ $\qquad y = \pm\sqrt{\frac{1}{9}}$

$y_1 = \frac{1}{3} \qquad y_2 = -\frac{1}{3}$

$x_1 = 10 \cdot \frac{1}{3} = \frac{10}{3}$

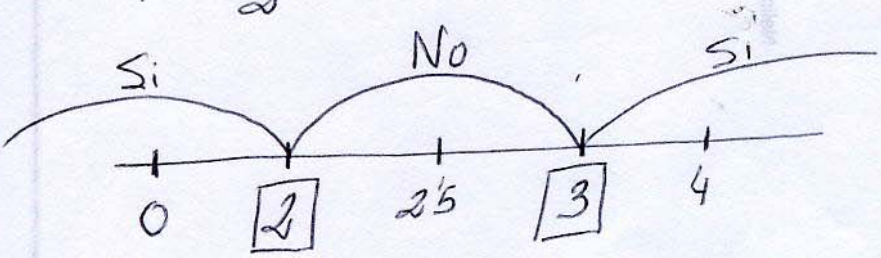
$x_2 = 10\left(-\frac{1}{3}\right) = -\frac{10}{3}$

③ $x^2 - 5x + 6 > 0$

$\downarrow$

$x^2 - 5x + 6 = 0 \rightarrow x = \frac{+5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{+5 \pm \sqrt{1}}{2}$

$x_1 = \frac{+5+1}{2} = 3 \qquad x_2 = \frac{5-1}{2} = 2$



$x^2 - 5x + 6 > 0$

$x = 0 \quad 0^2 - 5 \cdot 0 + 6 > 0$ Si

$x = 2'5 \quad 2'5^2 - 5 \cdot 2'5 + 6 \not> 0 \quad -0'25 \not> 0$ No

$x = 4 \quad 4^2 - 5 \cdot 4 + 6 > 0$ Si

Solución $(-\infty, 2) \cup (3, \infty)$

④

$2x + 3(x-1) \leq x+1$

$\frac{2x}{5} - \frac{x}{4} \geq \frac{2}{3}$

$\downarrow$

$\frac{24x - 15x}{60} \geq \frac{40}{60}$

$9x \geq 40$

$x \geq \frac{40}{9}$

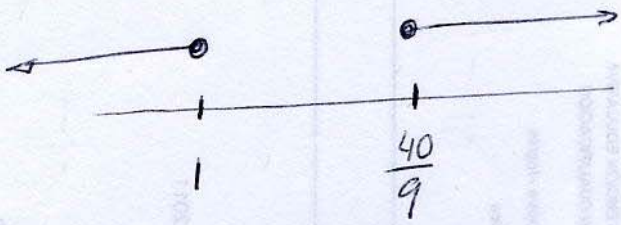
$\left[\frac{40}{9}, \infty\right)$

$2x + 3x - 3 \leq x + 1$

$2x + 3x - x \leq 1 + 3$

$4x \leq 4$

$\boxed{x \leq 1}$ $\quad (-\infty, 1]$



No tiene solución

(5)

$$\left.\begin{array}{l} 2x+16 \geq 2y \\ 2y-3x < 16 \end{array}\right\} \longrightarrow 2x+16 = 2y$$

$$y = \frac{2x+16}{2} = x+8$$

$y = x+8$

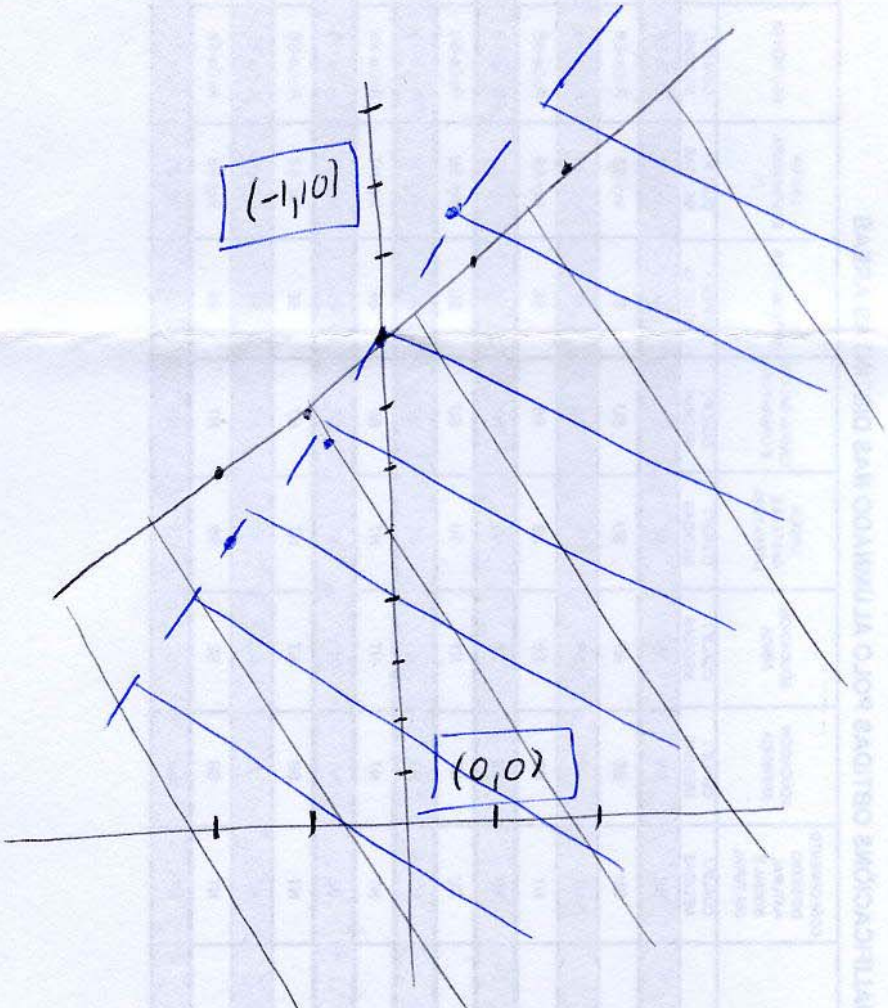
x	y
-2	+6
-1	+7
0	+8
1	+9
2	+10

$$2y - 3x = 16$$

$$2y = 16 + 3x$$

$$y = \frac{3x+16}{2}$$

x	y
-2	5
-1	13/2
0	8
1	19/2
2	11



$2x+16 \geq 2y$

$(0,0)$ $2\cdot 0 + 16 \geq 2\cdot 0$

$(-1,10)$ $2\cdot(-1)+16 \not\geq 2\cdot 10$

$2y - 3x < 16$

$(0,0)$ $2\cdot 0 - 3\cdot 0 < 16$ Si

$(-1,10)$ $2\cdot 10 - 3(-1) \not< 16$

(6) $2\log(4-x) = \log(3x+8) + \log(x+2)$

$\log(4-x)^2 = \log(3x+8)(x+2)$

$(4-x)^2 = (3x+8)(x+2)$

$16 + x^2 - 8x = 3x^2 + 6x + 8x + 16$

$x^2 - 3x^2 - 8x - 6x - 8x + 16 - 16 = 0$

$-2x^2 - 22x = 0$

$(-2x)(x+11) = 0$

$-2x = 0 \quad x_1 = 0$

$x+11 = 0 \quad x_2 = -11$

(7) $\cos\alpha = \frac{-\sqrt{3}}{2}$

$\operatorname{sen}\alpha = \frac{1}{2}$

$\operatorname{tag}\alpha = \frac{-\sqrt{3}}{3}$

$\sec\alpha = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$\operatorname{cosec}\alpha = 2$

$\operatorname{cotag}\alpha = -\sqrt{3}$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$\operatorname{sen}^2\alpha = 1 - \cos^2\alpha$

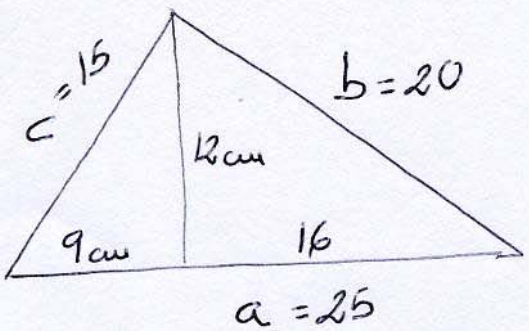
$\operatorname{sen}\alpha = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{1 - \frac{3}{4}}$

$\operatorname{sen}\alpha = \sqrt{\frac{4}{4} - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\operatorname{tag}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$\operatorname{tag}\alpha = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

(8)



Teorema altura

$h^2 = m \cdot n$

Teorema catetos

$b^2 = a \cdot m$

$c^2 = a \cdot n$

$h^2 = mn \rightarrow 12^2 = 9 \cdot m \quad m = \frac{12^2}{9} = 16 \text{ cm}$

$a = m + n = 9 + 16 = 25 \text{ cm}$

$b^2 = a \cdot m \rightarrow b = \sqrt{a \cdot m} = \sqrt{25 \cdot 16} = 20 \text{ cm}$

$c^2 = a \cdot n \rightarrow c = \sqrt{a \cdot n} = \sqrt{25 \cdot 9} = 15 \text{ cm}$