

Boletín Límites y Dominios III – Mat. 4º E.S.O.

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{5n} = \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \right]^5 = e^5$$

$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{3n+2} = \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{\frac{(3n+2)n}{n}} = \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \right]^{\frac{(3n+2)}{n}} = \left[\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n \right]^{\lim_{n \rightarrow +\infty} \frac{(3n+2)}{n}} = e^{\lim_{n \rightarrow +\infty} \frac{(3n+2)}{n}} = e^3$$

$$\lim_{n \rightarrow +\infty} \left(\frac{3n}{3n-7}\right)^{n^2} = \lim_{n \rightarrow +\infty} \left(\frac{3n-7+7}{3n-7}\right)^{n^2} = \lim_{n \rightarrow +\infty} \left(\frac{3n-7}{3n-7} + \frac{7}{3n-7}\right)^{n^2} = \lim_{n \rightarrow +\infty} \left(1 + \frac{7}{3n-7}\right)^{n^2} = \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{n^2} =$$

$$= \lim_{n \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{\frac{3n-7}{7} \cdot \frac{7}{3n-7} \cdot n^2} \right] = \lim_{n \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{\frac{3n-7}{7} \cdot \frac{7}{3n-7} \cdot n^2} \right] = \lim_{n \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{\frac{7n^2}{3n-7}} \right] =$$

$$= \lim_{n \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{\frac{3n-7}{7} \cdot \lim_{n \rightarrow +\infty} \frac{7n^2}{3n-7}} \right] = e^{\lim_{n \rightarrow +\infty} \frac{7n^2}{3n-7}} = e^{+\infty} \quad e = \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{\frac{3n-7}{7}}\right)^{\frac{3n-7}{7}} \quad \lim_{n \rightarrow +\infty} \frac{7n^2}{3n-7} = +\infty$$

A partir de los ejemplos anteriores calcula los siguientes límites.

a. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{5n}$

e. $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{3n+2}$

b. $\lim_{n \rightarrow +\infty} \left(\frac{n+5}{n+4}\right)^{n+4}$

f. $\lim_{n \rightarrow +\infty} \left(\frac{n+5}{n+4}\right)^{n+4}$

c. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n+5}\right)^{n+5}$

d. $\lim_{n \rightarrow \infty} \left(\frac{6n}{6n-5}\right)^{n^2}$